PyPSA Workshop

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Overview

How can (open) energy system modelling help?

Energy system modelling is about analysing technical feasibility, cost effectiveness and reliability of the system design & operation.

Such systems are complex:

- variability of wind, solar, demand
- grids, storage, EVs & industry

Ideally with **open-source** software & data for transparency, credibility & accessibility.

Current Energy Systems

- study impacts of price shocks
- study impacts of policy changes

Future Energy Systems

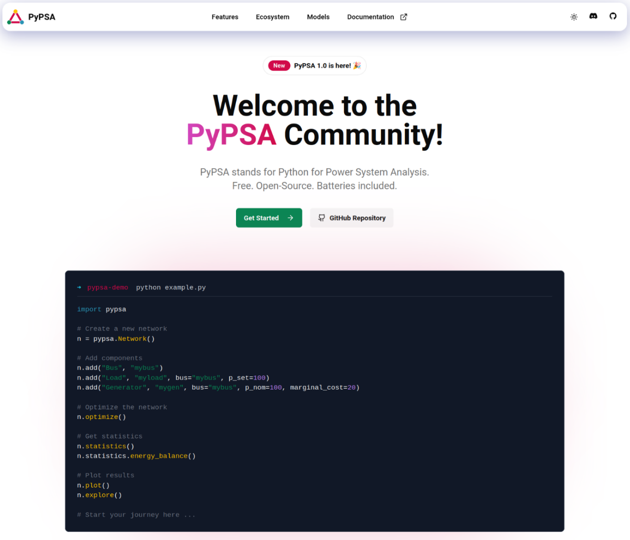
- transition to sustainable energy
- renewables integration

PyPSA Labs

What is PyPSA?

Our research focus:

- Cost-effective pathways to reduce greenhouse gas emissions while securing resilient energy supply.
- Evaluation of grid, renewables and storage expansion strategies; hydrogen strategies.
- Co-optimisation of generation, storage, conversion and transmission infrastructure.
- Algorithms to improve the tractability of models.
- All open source and open data for transparency and reproducibility.



pypsa.org

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Who else uses PyPSA?

Network operators

NGOs, universities & international organisations

Regulators

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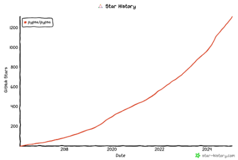
PyPSA:
Python for Power System Analysis



Capabilities

Capacity expansion (linear)
- single-horizon
- multi-horizon

Market modelling (linear)
- Linear optimal power flow
- N-1 security-constraints
- Unit commitment
- Security of supply analysis

Non-linear power flow
Modelling-to-Generate-Alternatives
Stochastic Optimisation

With components for

- Electricity transmission networks and pipelines
- Conventional generators with unit commitment constraints
- Variable generation with time series (e.g. wind, solar, hydro)
- Storage with efficiency losses and inflow/spillage for hydro
- Conversion between energy carriers (PtX, CHP, BEV, DAC)

Backend

- all data stored in pandas
- framework built for performance with large networks and time series
- interfaces to major solvers (Gurobi, CPLEX, HiGHS, Xpress), with linopy
- highly customisable, but no GUI

Economic Dispatch (ED)

Model short-term market dispatch with unit commitment, renewables, storage, multi-carrier conversion, and more.

Linear Optimal Power Flow (LOPF)

Optimize power flow in AC-DC networks using linearized KVL/KCL constraints with optional loss approximations.

Security-Constrained LOPF (SCLOPF)

Ensure system reliability by accounting for line outage contingencies under N-1 conditions.

Capacity Expansion Planning (CEP)

Plan long-term infrastructure investments for generation, storage, transmission, and conversion with continuous or discrete options.

Modular Design

Clean separation between data and modeling code enables flexible scenario development.

Pathway Planning

Co-optimize multi-period investments to design energy transition pathways with perfect foresight planning.

Rolling-Horizon Optimisation

Solve large-scale problems sequentially with myopic foresight and dynamic updates across time horizons.

Stochastic Optimisation

Two-stage stochastic programming for uncertainty handling with weighted scenarios and separate investment-dispatch decisions.

Modelling-to-Generate-Alternatives (MGA)

Explore alternative system designs with similar costs to understand trade-offs and decision flexibility.

Performance

Designed to scale with high-resolution networks, minimizing memory usage and solver overhead.

Policy Constraints

Built-in support for CO_2 limits, subsidies, resource limits, expansion limits, and growth constraints.

Custom Constraints

Impose custom objectives, variables and constraints using Linopy for specialized requirements.

Solver Flexibility

Support for wide range of LP, MILP, and QP solvers from open-source to commercial solutions.

Data Backbone

Uses pandas for data handling and linopy for optimization with efficient solver interfacing.

Resolution Control

Flexible control over temporal, spatial, and sectoral scope and detail for any analysis needs.

Sector-Coupling

Model integrated multi-carrier energy systems with heat pumps, electrolysers, EVs, and synthetic fuels.

Standard Grid Components

Includes standard types for lines and transformers from pandapower for realistic grid modeling.

Static Power Flow Analysis

Compute non-linear and linearised load flows for AC-DC grids using Newton-Raphson method.

Visualisations

Plot statistics, time series, spatial distributions of line loadings and dispatch decisions.

Scalability

Handle models from small prototypes to continent-scale systems on high-performance clusters.

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PyPSA Features Ecosystem Models Roadmap Events Docs

What's next

Roadmap

What we're planning to build and a selection of what was recently shipped. If you want to get involved, reach out to us or follow along in the repositories.

Planned — **Stricter network validation**
Catch inconsistent inputs for better and earlier user feedback.

Planned — **New file format**
A new network format, faster than today's netCDF, that loads data lazily, reads only what is needed and supports streaming.

Planned — **Improved model scaling**
Automatic scaling to improve the model's numerical conditioning, solver stability and speed.

Planned — **Custom constraint interface**
Define constraints and models in a text-based format, without the need for scripting.

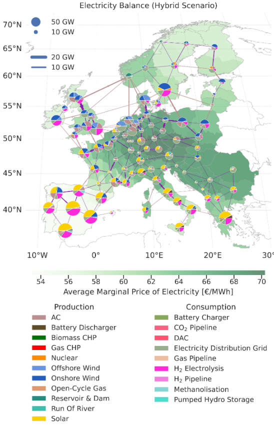
Planned — **Roundtrip Serialisation IO**
Any network object can be saved and loaded back identically, with no state lost, including custom constraints, solver metadata and geospatial data.

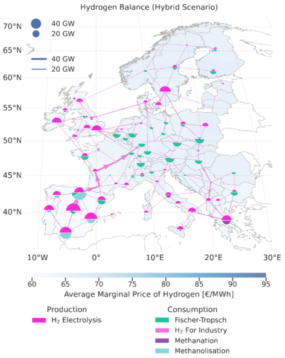
Planned — **Recourse investments**
Allow certain investment variables to adjust per scenario as better information about uncertain conditions becomes available, extending recourse beyond dispatch decisions.

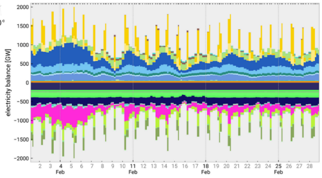
PyPSA-Eur: A sector-coupled open model of the European energy system

Automated workflow to build energy system model of Europe from raw open data with high spatial and temporal resolution:

1. OSM transmission lines (>220 kV) + TYNDP
2. a database of existing power plants,
3. time series for electricity demand,
4. time series for wind/solar availability, and
5. geographic wind/solar potentials
6. cost and efficiency assumptions
7. methods for model simplification
8. more for sector-coupled networks like pipelines, LNG terminals, electric vehicles, industry locations,







Least-cost optimisation of dispatch and investments

Co-optimise generation, storage, conversion and transmission infrastructure:

$$\text{Min}\left[\begin{array}{c}\textbf{Yearly}\\\textbf{system}\\\textbf{costs}\end{array}\right] = \text{Min}\left[\sum_r \left(\begin{array}{c}\textbf{Annualised}\\\textbf{capital}\\\textbf{costs}\end{array}\right) + \sum_{r,t}\left(\begin{array}{c}\textbf{Operating}\\\textbf{costs}\end{array}\right)\right]$$

subject to *(selection)*

- meeting energy demand in each region r and time t
- transmission constraints between regions with (linearised) power flow
- geographical potentials for technologies (e.g. wind and solar) $\forall\, r$
- wind, solar, hydro availability time series $\forall\, r, t$
- emission reduction targets & reserve margin requirements

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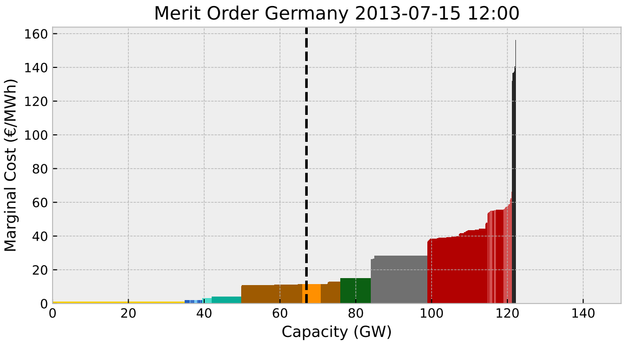
Dispatch Optimisation

Electricity Markets as Optimisation Problems

Dispatch optimisation concerns which power plants should produce at what time to meet demand at least cost, subject to technical constraints.

For one market region and one hour we can start with the **merit order**, but reality is more complex → use mathematical optimisation.



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Electricity Markets as Optimisation Problems

For a single region and hour, we have s generators with marginal costs o_s (€/MWh) and (available) capacities G_s (MW). We seek to minimise the operational costs of generator dispatch g_s to fulfil the electricity load of d MW. The **optimisation problem**:

$$\min_{g_s} \sum_s o_s g_s$$

such that

$$g_s \leq G_s$$

$$g_s \geq 0$$

$$\sum_s g_s = d$$

(1) is the **objective function**, (2 - 4) are inequality and equality **constraints**. g_s are the **optimisation variables** or **decision variables**. G_s, o_s and d are **input parameters**.

Adding a regional component $i \rightarrow$ multiple zones

$$\min_{g_{i,s}, f_\ell} \sum_{i,s} o_{i,s} g_{i,s}$$

such that

$$0 \leq g_{i,s} \leq G_{i,s}$$

$$\sum_s g_{i,s} - \sum_\ell K_{i\ell} f_\ell = d_i \qquad \text{Kirchhoff Current Law}$$

$$|f_\ell| \leq F_\ell \qquad \text{line limits}$$

$$\sum_\ell C_{\ell c} x_\ell f_\ell = 0 \qquad \text{Kirchhoff Voltage Law}$$

where f_ℓ is the flow (e.g. into/from other zones), $K_{i\ell}$ is the incidence matrix and F_ℓ is the line capacity, x_ℓ is the reactance, and $C_{\ell c}$ is the cycle matrix.

Adding a temporal component $t \rightarrow$ multiple snapshots

$$\min_{g_{i,s,t}, f_{\ell,t}} \sum_{i,s,t} o_{i,s} g_{i,s,t}$$

such that

$$0 \leq g_{i,s,t} \leq \hat{g}_{i,s,t} G_{i,s}$$

$$\sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} = d_{i,t} \qquad \text{Kirchhoff Current Law}$$

$$|f_{\ell,t}| \leq F_\ell \qquad \text{line limits}$$

$$\sum_\ell C_{\ell c} x_\ell f_{\ell,t} = 0 \qquad \text{Kirchhoff Voltage Law}$$

where $\hat{g}_{i,s,t}$ can, for instance, represent the capacity factor of a solar panel.

Adding a temporal component $t \rightarrow$ with storage

Storage units r, like batteries, can charge/buy when prices are low, discharge/sell when prices are high:

Add constraints where storage can dispatch power within its **discharging capacity**:

$$0 \leq g_{i,r,t,\text{discharge}} \leq G_{i,r,\text{discharge}}$$

and consume power to charge the storage within its **charging capacity**:

$$0 \leq g_{i,r,t,\text{charge}} \leq G_{i,r,\text{charge}}$$

Also add a constraint on the limit $E_{i,r}$ on the **state of charge** $e_{i,r,t}$:

$$0 \leq e_{i,r,t} \leq E_{i,r}$$

The dispatch decisions are related by the following constraint:

$$e_{i,r,t} = \eta^{\text{self-discharge}}_{i,r,t} e_{i,r,t-1} + \eta^{\text{charge}}_{i,r} g_{i,r,t,\text{charge}} - \frac{1}{\eta^{\text{discharge}}_{i,r}} g_{i,r,t,\text{discharge}}$$

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Interpreting shadow prices

Shadow prices (dual variables) measure the marginal change in the objective value of the optimal solution by relaxing the constraint by a small amount.

$$\min_{g_s} \sum_s o_s g_s \qquad s.t.$$

$$\begin{aligned} g_s &\leq G_s & \leftrightarrow \bar{\mu}_s \\ -g_s &\leq 0 & \leftrightarrow \underline{\mu}_s \\ \sum_s g_s &= d & \leftrightarrow \lambda \end{aligned}$$

The **dual variable** λ corresponds to the **market clearing price**, i.e. :

- the marginal cost of the marginal generator
- the cost of consuming an additional unit of power

Long-Term Planning

From modelling electricity markets to system planning

The dispatch optimisation only looks at the **short-term perspective**, i.e. how to make the best use of the existing infrastructure.

Degrees of freedom:

1. Dispatch of power plants
2. Operation of storage units
3. Curtailment of renewables
4. Investment in new generation
5. Investment in new storage units
6. Transmission reinforcement

Now, we will also look at the **long-term perspective**. We will optimise investment in the capacities of generators, storage and transmission by promoting the capacities, like G and F, to decision variables.

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Example: Toy Model of Capacity Expansion Planning

Build your own zero-emission electricity supply

Introduction

This tool calculates the cost of meeting a constant electricity demand from a combination of wind power, solar power and storage for different regions of the world.

First choose your location to determine the weather data for the wind and solar generation. Then choose your cost and technology assumptions to find the solution with least cost. Storage options are batteries and hydrogen from electrolysis of water.

Fun things to try out:

- remove technologies with the checkboxes, e.g. hydrogen gas storage or wind, and see system costs rise
- set solar or battery costs very low, to simulate breakthroughs in manufacturing

See also this Twitter thread for an overview of the model's features and capabilities.

This is a **toy model** with a **strongly simplified** setup. Please read the warnings below.

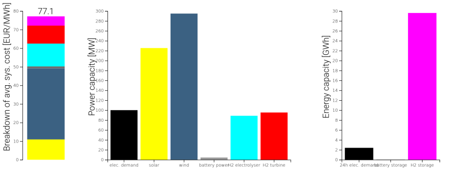
Step 1: Select location and weather year

- ◉ Select country
- ○ Select state or province (for Australia, China, Germany, India, Russia and United States)
- ○ Select point, rectangle or polygon, using the toolbox that appears at top-right

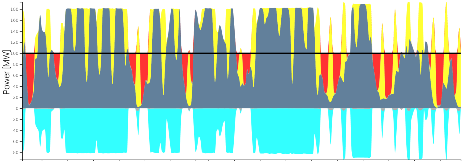
Average system cost [EUR/MWh]: 77.1

Cost is per unit of energy delivered in 2015 euros. For comparison, household electricity rates (including taxes and grid charges) averaged 211 EUR/MWh in the European Union in 2018 & 132 USD/MWh in the United States in 2019

Average marginal price of hydrogen [EUR/MWh LHV]: 78.8, [EUR/kg]: 2.60



Dispatch over a year to meet the constant demand (electricity demand is the black line; you can zoom and pan to see the details; negative values correspond to storage consuming electricity):



Brown, T. `https://model.energy/`; optimises capacity and dispatch for single-node model with wind, solar, batteries and hydrogen storage for a constant demand.

Electricity system operation as optimisation problem

Decision variables (all operational) are shown in red.
Capacity expansion is an extension of this problem!

$$\min_{g,e,f} \sum_{i,s,t} o_{i,s} g_{i,s,t} \quad \text{s.t.}$$

$$d_{i,t} = \sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} \qquad \text{energy balance}$$

$$0 \leq g_{i,s,t} \leq \hat{g}_{i,s,t} G_{i,s} \qquad \text{generator limits}$$

$$0 \leq g_{i,r,t,\text{discharge}} \leq G_{i,r,\text{discharge}} \qquad \text{storage discharge limits}$$

$$0 \leq g_{i,r,t,\text{charge}} \leq G_{i,r,\text{charge}} \qquad \text{storage charge limits}$$

$$0 \leq e_{i,r,t} \leq E_{i,r} \qquad \text{storage energy limits}$$

$$e_{i,r,t} = \eta^0_{i,r,t} e_{i,r,t-1} + \eta^1_r g_{i,r,t,\text{charge}} - \frac{1}{\eta^2_r} g_{i,r,t,\text{discharge}} \qquad \text{storage consistency}$$

$$-F_\ell \leq f_{\ell,t} \leq F_\ell \qquad \text{line limits}$$

$$0 = \sum_\ell C_{\ell c} x_\ell f_{\ell,t} \qquad \text{Kirchhoff's Voltage Law (KVL)}$$

Electricity system planning as optimisation problem

In addition to previous operational decision variables, we convert the parameters for capacities into investment decision variables with new associated parameters.

$$\min_{g,e,f,G,E,F} \sum_{i,s,t} w_t o_s g_{i,s,t} + \sum_{i,s} c_s G_{i,s} + c_{r,\text{dis/charge}} G_{i,r,\text{dis/charge}} + c_r E_{i,r} + c_\ell F_\ell \quad \text{s.t.}$$

$$d_{i,t} = \sum_s g_{i,s,t} - \sum_\ell K_{i\ell} f_{\ell,t} \qquad \text{energy balance}$$

$$0 \leq g_{i,s,t} \leq \hat{g}_{i,s,t} G_{i,s} \qquad \text{generator limits}$$

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$$0 = \sum_\ell C_{\ell c} x_\ell f_{\ell,t} \qquad \text{Kirchhoff's Voltage Law (KVL)}$$

Changes in objective function with capacity expansion.

$$\min_{g,e,f,G,E,F} \sum_{i,s,t} w_t o_s g_{i,s,t} + \sum_{i,s} c_s G_{i,s} + c_{r,\text{dis/charge}} G_{i,r,\text{dis/charge}} + c_r E_{i,r} + c_\ell F_\ell$$

- We have **marginal cost** o_s for each generator (€/MWh),
- we now also have **specific capital costs** $c_\star$ (e.g. building new plant, €/MW),
- and a weighting w_t denoting how much time a time step t represents.

Units of objective function with capacity expansion

$$\min \quad \overbrace{\sum_{i,s,t} \mathrm{h} \cdot \frac{€}{\mathrm{MWh}} \cdot \mathrm{MW}}^{\text{operational costs}} + \overbrace{\sum_{i,s} \frac{€}{MW} \cdot \mathrm{MW} + \frac{€}{MWh} \cdot \mathrm{MWh}}^{\text{investment costs}}$$

- For a fair comparison between operational costs and investments, we need to align the terms such that they **represent the same time frame**.
- **One common option: yearly system costs (€/a)**
  - operational: one year of representative operational conditions ($\sum_t w_t = 8760$h)
  - investment: annualised investment costs ($c_\star$ in €/MW/a = €/MW/8760h)

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Uncertainty and Risk in Energy System Planning

Start: The model solves a capacity expansion model

In **stylised** form, a **linear optimisation problem (LP)** si solved for least-cost investment and dispatch:

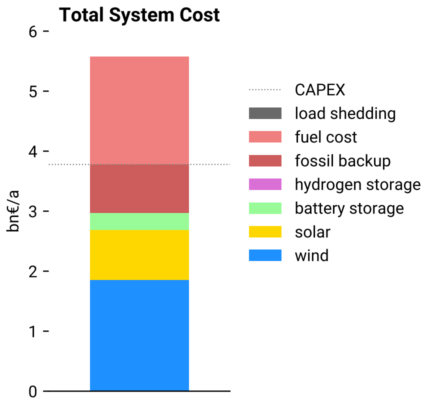
$$\min \quad \text{CAPEX} + \sum_t \text{OPEX}_t$$

$$\text{s.t.} \quad \text{dispatch}_{\text{tech},t} \leq \text{capacity}_{\text{tech}} \quad \forall\, t, \text{tech}$$

$$\sum_{\text{tech}} \text{dispatch}_{\text{tech},t} = \text{demand}_t \quad \forall\, t$$

...plus renewable availability, storage operations & network constraints across all hours in the year.

Single-node model.energy scenario with fossil backup at → 100 €/MWh$_e$ with demand & renewable profiles in EU country



The following toy models are all implemented in PyPSA v1.1 and solved with HiGHS; Brown, T., Hörsch, J. & Schlachtberger, D. PyPSA: Python for Power System Analysis. *Journal of Open Research Software* **6**, 4. doi:`10.5334/jors.188` (2018)

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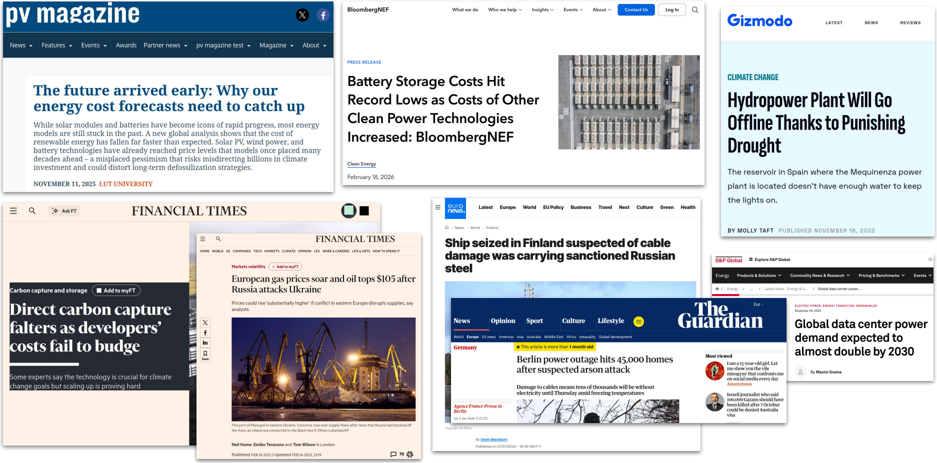
Our era of compounding uncertainties



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The future arrived early: Why our energy cost forecasts need to catch up

While solar modules and batteries have become icons of rapid progress, most energy models are still stuck in the past. A new global analysis shows that the cost of renewable energy has fallen far faster than expected. Solar PV, wind power, and battery technologies have already reached price levels that models once placed many decades ahead – a misplaced pessimism that risks misdirecting billions in climate investment and could distort long-term defossilization strategies.

NOVEMBER 11, 2025 LUT UNIVERSITY

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PRESS RELEASE

Battery Storage Costs Hit Record Lows as Costs of Other Clean Power Technologies Increased: BloombergNEF

Clean Energy

February 18, 2026

Gizmodo

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BY MOLLY TAFT PUBLISHED NOVEMBER 18, 2022

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Agence France-Presse in Berlin

S&P Global

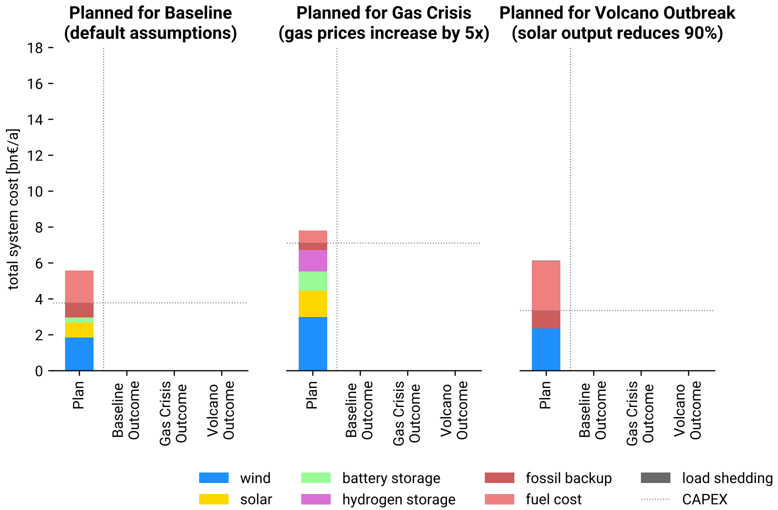
ELECTRIC POWER, ENERGY TRANSITION, RENEWABLES

Global data center power demand expected to almost double by 2030

By Maxim Grama

PV Magazine; BloombergNEF; Financial Times;
The Guardian; S&P Global; euronews; Gizmodo

At least, we could try a few more scenarios, right?



We obtain different strategies per scenario:

- renewables and storage for gas crisis
- less solar & more wind for volcano outbreak

How to decide which strategy to implement?

What happens if the wrong scenario arrives?

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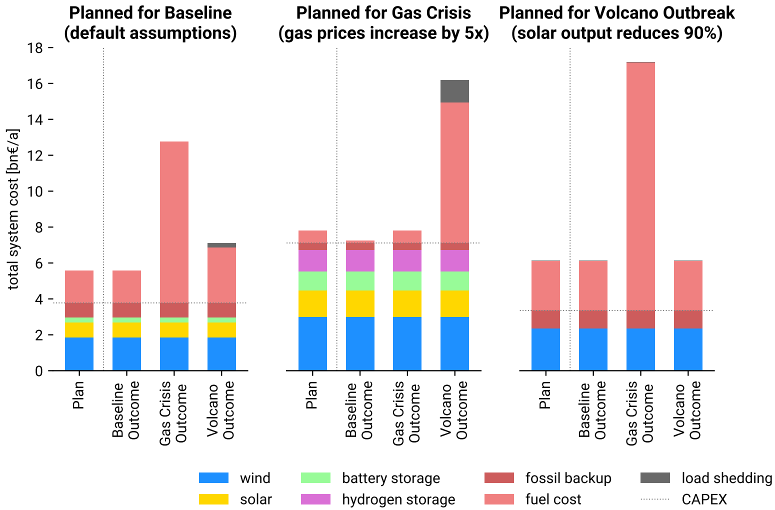
But every plan has an Achilles heel.



All strategies perform poorly in at least one scenario they were not planned for.

Let's assume the following probabilities:

Baseline:	50%
Gas Crisis:	40%
Volcano:	10%

Which strategy would you choose and why?

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The Regret Matrix

Regret
(% of perfect knowledge solution)

Planned for \ Outcome	Baseline	Gas Crisis	Volcano
Baseline	+0	+129	+28
Gas Crisis	-7	+0	+107
Volcano	+0	+181	+0

Expectation [bn€/a]

Another common way to show performance across scenarios is the **regret matrix**.

It measures the extra cost of choosing a sub-optimal investment plan for the scenario s that actually occurs:

$$\text{regret}_s(\text{plan}) = \text{cost}_s(\text{plan}) - \text{cost}_s(\text{optimal plan for } s)$$

Regret matrices can also be expressed in relative terms, i.e. as a percentage of the optimal cost.

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The Regret Matrix

Regret
(% of perfect knowledge solution)

Planned for	Outcome: Baseline	Outcome: Gas Crisis	Outcome: Volcano	Expectation [bn€/a]
Baseline	+0	+129	+28	8.60
Gas Crisis	-7	+0	+107	8.37
Volcano	+0	+181	+0	10.55

Knowing the probabilities, we can calculate the **expected total cost** of each investment plan.

But uncertainty is <u>not considered</u> in planning.

Risk of high costs present in all plans!

How to incorporate uncertainty into planning?

Method: Stochastic Programming

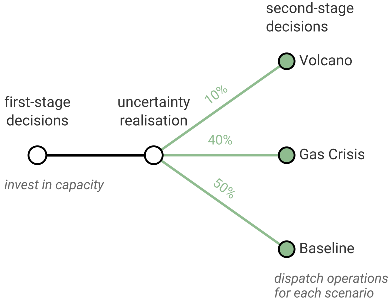
Two-stage stochastic programming for minimising expected total cost across all scenarios:

$$\min \quad \overbrace{\text{CAPEX}}^{\substack{\text{here-and-now} \\ \text{first-stage decisions}}} + \sum_{t,s} p_s \cdot \overbrace{\text{OPEX}_{t,s}}^{\substack{\text{wait-and-see} \\ \text{second-stage decisions}}}$$

$$\text{s.t.} \quad \text{dispatch}_{\text{tech},t,s} \leq \text{capacity}_{\text{tech}} \quad \forall t, \text{tech}, s$$

$$\sum_{\text{tech}} \text{dispatch}_{\text{tech},t,s} = \text{demand}_{t,s} \quad \forall t, s$$

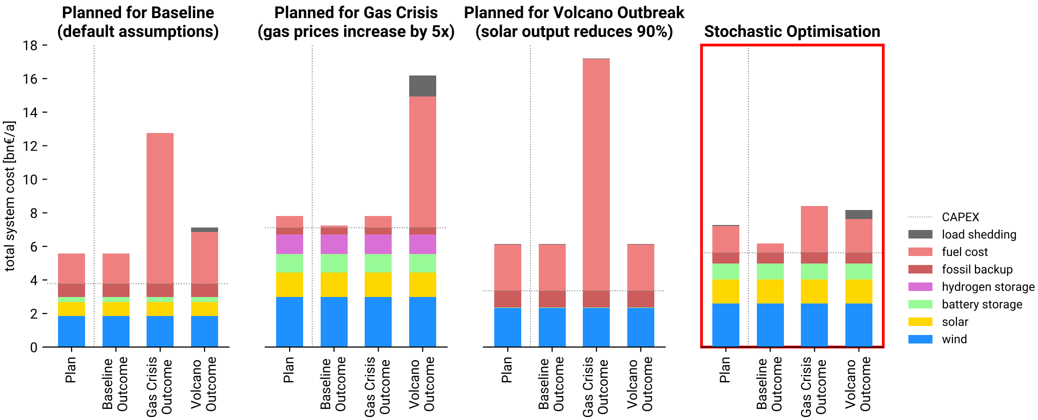
where p_s is the probability of scenario s.



Investment decisions are fixed.
Operational decisions can adapt.

Birge, J. R. & Louveaux, F. *Introduction to Stochastic Programming.* 2nd ed. doi:10.1007/978-1-4614-0237-4 (Springer, 2011)

Stochastic plan hedges across all scenarios.



Stochastic plan is not optimal for any single scenario – but best plan **hedging** across them.

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Let’s complete our regret matrix

Regret
(% of perfect knowledge solution)

Planned for	Outcome: Baseline	Outcome: Gas Crisis	Outcome: Volcano	Expectation [bn€/a]
Baseline	+0	+129	+28	8.60
Gas Crisis	-7	+0	+107	8.37
Volcano	+0	+181	+0	10.55
Stochastic	+11	+8	+33	7.26

Stochastic plan reduces expected costs by ≈**13%**.

However, it still has substantial risk in the **low-probability** volcano outbreak event.

Reason: By minimising expected costs, the stochastic plan is **risk-neutral** – even though investors and planners are **risk-averse**.

How to incorporate risk-aversion into planning?

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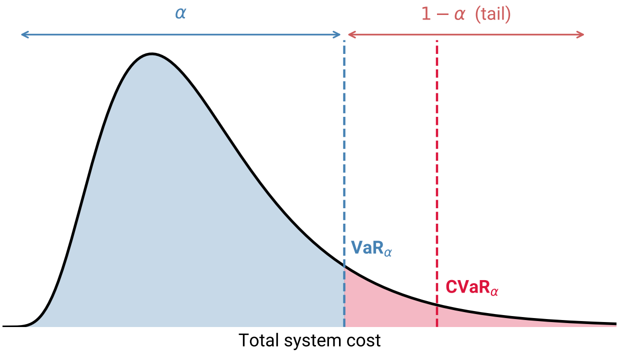
Method: Conditional Value at Risk (CVaR)



CVaR is a risk measure (from finance) that captures the expected cost in the worst-case tail:

$$\mathrm{CVaR}_\alpha = \mathbb{E}[\mathrm{OPEX}_s \mid \mathrm{OPEX}_s \geq \mathrm{VaR}_\alpha]$$

VaR$_\alpha$: cost at the α-quantile & **threshold** for worst-case tail.

Users choose α to control emphasis on **extreme** scenarios.

Rockafellar, R. T. & Uryasev, S. Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking & Finance* **26**, 1443–1471. doi:10.1016/S0378-4266(02)00271-6 (2002)

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Method: Risk-Averse Stochastic Programming with CVaR

Risk-averse planners accept higher expected cost to avoid worst-case outcomes.

With CVaR in objective, parameter ω shifts weight from expected cost to tail cost:

$$\min \quad \underbrace{\text{CAPEX}}_{\text{common investments}} + \underbrace{(1-\omega)\sum_{t,s} p_s \cdot \text{OPEX}_{t,s}}_{\text{risk-neutral expected cost fraction}} + \underbrace{\omega \cdot \text{CVaR}_\alpha}_{\text{risk-averse tail cost fraction}}$$

$$\text{s.t.} \quad \text{CVaR}_\alpha = \mathbb{E}[\text{OPEX}_{t,s} \mid \text{OPEX}_{t,s} \geq \text{VaR}_\alpha]$$

$$\text{dispatch}_{\text{tech},t,s} \leq \text{capacity}_{\text{tech}} \quad \forall\, t, \text{tech}, s$$

$$\sum_{\text{tech}} \text{dispatch}_{\text{tech},t,s} = \text{demand}_{t,s} \quad \forall\, t, s$$

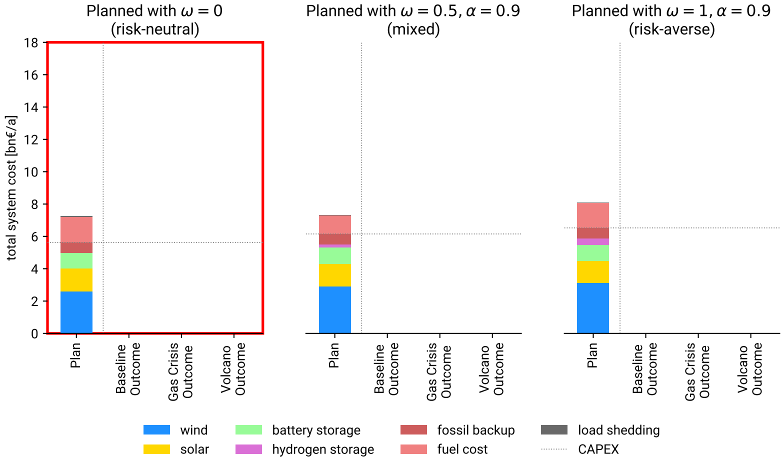
Users control **risk-aversion** with ω.

Worst case is **internally identified**.

Can be formulated as **LP**.

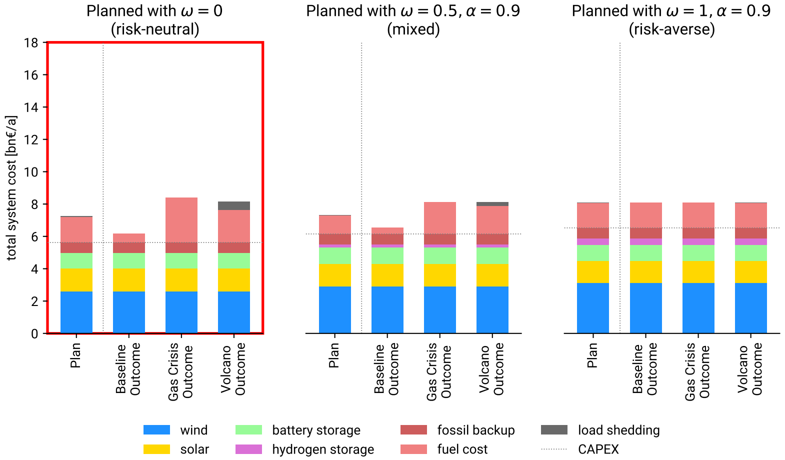
Rockafellar, R. T. & Uryasev, S. Conditional Value-at-Risk for General Loss Distributions. *Journal of Banking & Finance* **26**, 1443–1471. doi:10.1016/S0378-4266(02)00271-6 (2002)

How varying risk-aversion levels affect investments



Higher risk aversion (ω) hedges against risky scenarios by **shifting costs** from the tail (OPEX) to upfront investments (CAPEX).

How varying risk-aversion levels affect investments



Increasing ω progressively **levels out differences** in cost across scenarios, reducing risk.

This comes at the cost of higher expected costs.

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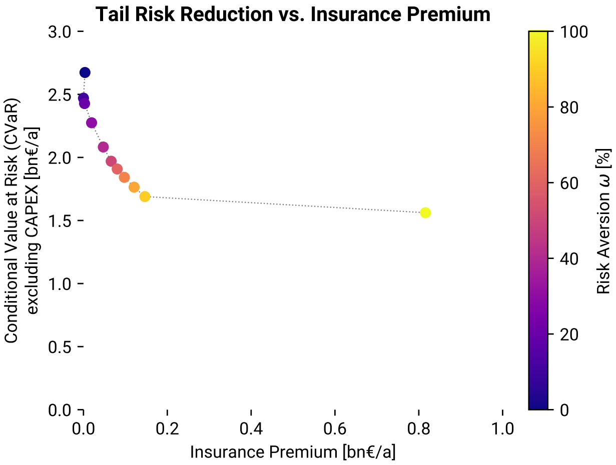
Risk-Hedging: Trading Expected Cost for Resilience



Insurance Premium

Additional expected cost of applying a hedging strategy to reduce risk.

Tail Risk Reduction

Decrease in CVaR, i.e. risk reduction achieved by the hedging strategy.

Summary

1. Deterministic single-scenario plans are **fragile**
   — each optimal for one world, costly in others

2. **Stochastic optimisation** finds plans hedging across uncertain futures
   — it minimises expected costs and is risk-neutral

3. **CVaR** embeds risk preferences and suppresses tail costs
   — investors willing to pay more upfront to avoid worst-case outcomes

4. The **cost-risk frontier** makes trade-offs navigable

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Backup Material

Backup: Present Value

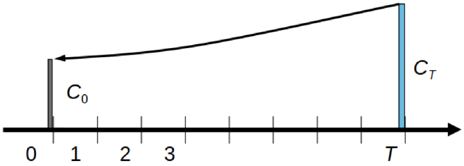
To compare income and costs in different years, we need to agree on a particular point in time; e.g. today's value, known as **present value**.

For an **interest rate** r, we multiply the cash flow in t years by the discount factor

$$\frac{1}{(1+r)^t}$$

to calculate the present value. We have **discounted** the future cash flow.

Future income or costs are **worth less** from today's point of view (if $r > 0$).



  Tom Brown

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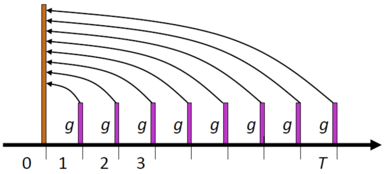
Backup: Present value factor for repeating cashflows

If we have a repeating constant cashflow CF for T years, the present value is:

$$PV = CF \cdot \sum_{t=1}^{T} \frac{1}{(1+r)^t}$$

The sum $\sum$ is called the **Present Value Factor** $PVF(r, T)$, which brings a series of equal cashflows (pink) to the present value (orange):



For a derivation of the present value factor see `https://nworbmot.org/courses/ee-24/ee-6-financial_management.pdf`, slide 21.

Backup: Annuity factor to distribute upfront investments

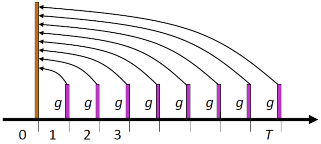
Vice versa, we can use the present value factor $PVF(r, T)$ to convert an initial investment I to identical annual payments to compare them with the yearly cashflow:

$$A = \frac{I}{PVF(r,T)}$$

A is also called the **annuity**, i.e. the annualised investment costs.

The **annuity factor** $a(r, T)$ spreads the capital cost I evenly over the operational years of the investment while taking account of the interest rate / discount rate r:

$$a(r,T) = \frac{1}{PVF(r,T)} = \frac{r}{1-(1+r)^{-T}}$$



reversed!